Papers written by
Australian Maths
Software

SEMESTER TWO

MATHEMATICS SPECIALIST

UNITS 3-4

2017

SOLUTIONS

Section One

1. (3 marks)

$x^2 + xy + y^2 = 3,$

$2x + 1 \times y + \frac{dy}{dx} \times x + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(x + 2y) = -2x - y$

$\frac{dy}{dx} = \frac{-2x - y}{x + 2y}$ ✓

$If\ x = 1,\ y = ?$

$1 + y + y^2 = 3$

$y^2 + y - 2 = 0$

$(y + 2)(y - 1) = 0$

$y = -2\ \ or\ \ y = 1$ ✓

$(1,-2)\ \ or\ \ (1,1)$

$At\ (1,-2)\ \frac{dy}{dx} = \frac{-2+2}{1-4} = 0 \quad At\ (1,1)\ \frac{dy}{dx} = \frac{-3}{3} = -1$ ✓

(3)

2. (12 marks)

(a) $\int_{\pi/6}^{\pi/3} \left(cos^2(x) - sin^2(x)\right)\, dx$

$= \int_{\pi/6}^{\pi/3} \left(cos(2x)\right)\, dx$ ✓

$= \left[\frac{sin(2x)}{2}\right]_{\pi/6}^{\pi/3}$

$= \frac{1}{2}\left(sin\left(\frac{2\pi}{3}\right) - sin\left(\frac{\pi}{3}\right)\right)$ ✓

$= \frac{1}{2}\left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right)$

$= 0$ ✓

(3)

(b) $\int_0^{0.5} \left(e^x - e^{-x}\right)^2 \, dx$

$$= \int_0^{0.5} \left(e^{2x} - 2 + e^{-2x}\right) dx$$

$$= \left[\frac{e^{2x}}{2} - 2x + \frac{e^{-2x}}{-2}\right]_0^{0.5} \quad ✓$$

$$= \left(\frac{e^1}{2} - 1 - \frac{e^{-1}}{2}\right) - \left(\frac{e^0}{2} - 0 - \frac{e^0}{2}\right)$$

$$= \frac{e}{2} - 1 - \frac{1}{2e} \quad ✓$$

(2)

(c) $\int \frac{2x-1}{x^2-x-6} dx$

$$\frac{2x-1}{x^2-x-6} = \frac{2x-1}{(x-3)(x+2)}$$

$$= \frac{a}{(x-3)} + \frac{b}{(x+2)} \quad ✓$$

$$= \frac{a(x+2)+b(x-3)}{(x-3)(x+2)}$$

$$= \frac{x(a+b)+(2a-3b)}{(x-3)(x+2)}$$

$\therefore \quad a+b=2 \quad 1.$

$2a-3b=-1 \quad 2.$ ✓

Subs $a = 2-b$ into 2.

$2(2-b)-3b=-1$

$5b=5$

$b=1 \quad \therefore\ a=1$

$$\therefore \int \frac{2x-1}{x^2-x-6} dx = \int \frac{1}{(x-3)} + \frac{1}{(x+2)} dx \quad ✓$$

$$= ln(x-3) + ln(x+2) + c \quad ✓$$

$$= ln(x-3)(x+2) + c$$

$$= ln\left(x^2-x-6\right) + c$$

(4)

(d) $\int \frac{(ln(x))^3}{x} dx$ using the substitution $u = ln(x)$

$$\frac{du}{dx} = \frac{1}{x}$$

$$du = \frac{dx}{x} \quad ✓$$

$$\equiv \int u^3 du \quad ✓$$

$$= \frac{u^4}{4} + c$$

$$\equiv \frac{(ln(x))^4}{4} + c \quad ✓$$

(3)

3. (3 marks)

Express $\sqrt{1+\sqrt{3}i}$ in cis form

Let $1+\sqrt{3}i = r\,cis(\theta)$

$$\left|1+\sqrt{3}i\right| = \sqrt{1^2 + \left(\sqrt{3}\right)^2} = 2 \quad ✓$$

$$tan(\theta) = \frac{\sqrt{3}}{1}$$

$$\theta = \frac{\pi}{3} \quad ✓$$

$$\therefore\ 1+\sqrt{3}i = 2cis\left(\frac{\pi}{3}\right)$$

$$\sqrt{1+\sqrt{3}i} = \sqrt{2}\left(cis\left(\frac{\pi}{3}\right)\right)^{1/2}$$

$$= \sqrt{2}cis\left(\frac{\pi}{6}\right) \quad ✓$$

Therefore $\sqrt{1+\sqrt{3}i} = \sqrt{2}cis\left(\frac{\pi}{6}\right)$

(4)

4. (14 marks)

(a) (i) $\dfrac{(1+2i)(3-i)^2}{10(2+i)}$

$=\dfrac{(1+2i)(9-6i+i^2)}{10(2+i)}\times\dfrac{(2-i)}{(2-i)}$ ✓

$=\dfrac{(1+2i)(8-6i)}{10(4-i^2)}\times(2-i)$

$=\dfrac{2(4-3i)(2+3i-2i^2)}{10\times 5}$ ✓

$=\dfrac{2(4-3i)(4+3i)}{50}$ ✓

$=\dfrac{(16-9i^2)}{25}$

$=\dfrac{25}{25}$

$=1$ ✓

(4)

(ii) $\dfrac{cis(40^o)\left(cis(10^o)\right)^2}{cis(120^o)}$

$=cis(40^o)\left(cis(20^o)\right)cis(-120^o)$ ✓

$=cis(60^o-120^o)$

$=cis(-60^o)$ ✓

$=cos(-60^o)+i\,sin(-60^o)$

$=\dfrac{1}{2}-\dfrac{i\sqrt{3}}{2}$ ✓

(3)

(b) Solve for z $z^3-4z^2+14z-20=0$.

$Let\ P(z)=z^3-4z^2+14z-20$

$P(2)=8-16+28-20$

$P(2)=0$

$\therefore z=2$ is a solution ✓

$$\begin{array}{r|rrrr} 2 & 1 & -4 & 14 & -20 \\ & \downarrow & 2 & -4 & 20 \\ \hline & 1 & -2 & 10 & 0 \end{array} \quad \checkmark$$

$\therefore P(z) = (z-2)(z^2 - 2z + 10)$

$z = 2$ or $z^2 - 2z + 10 = 0$

$$z = \frac{2 \pm \sqrt{4-40}}{2} \quad -36 = 36i^2$$

$$z = \frac{2 \pm 6i}{2}$$

$z = 2$ or $z = 1 \pm 3i$ ✓

(3)

(c) Prove the identity $sin(4x) = 8\,sin(x)cos^3(x) - 4\,sin(x)cos(x)$ using De Moivre's theorem.

$cis(4x) = (cis(x))^4$

$= (cos(x) + i\,sin(x))^4$ ✓

$= cos^4(x) + 4cos^3(x)i\,sin(x) + 6cos^2(x)i^2\,sin^2(x) + 4cos(x)i^3\,sin^3(x) + i^4\,sin^4(x)$

$cis(4x) = cos^4(x) + 4i\,cos^3(x)sin(x) - 6cos^2(x)sin^2(x) - 4i\,cos(x)sin^3(x) + sin^4(x)$ ✓

$cos(4x) + i\,sin(4x) = \left[cos^4(x) - 6cos^2(x)sin^2(x) + sin^4(x)\right] + i\left[4cos^3(x)sin(x) - 4cos(x)sin^3(x)\right]$

$\therefore\ sin(4x) = \left[4cos^3(x)sin(x) - 4cos(x)sin^3(x)\right]$ ✓

$= 4cos^3(x)sin(x) - 4cos(x)sin(x)(1 - cos^2(x))$

$= 4cos^3(x)sin(x) - 4cos(x)sin(x) + 4cos^3(x)sin(x)$ ✓

$\therefore\ sin(4x) = 8\,sin(x)cos^3(x) - 4\,sin(x)cos(x)$

(4)

5. (7 marks)

(a) $\boldsymbol{r}(t) = (1 + sin(t))\boldsymbol{i} + (1 - cos(t))\boldsymbol{j}$

(i) Show that $(x-1)^2 + (y-1)^2 = 1$.

$x = 1 + sin(t) \qquad y = 1 - cos(t)$ ✓

$sin(t) = x - 1 \qquad cos(t) = 1 - y$

$sin^2(t) + cos^2(t) = 1$

$(x-1)^2 + (1-y)^2 = 1 \quad but \quad (1-y)^2 = (y-1)^2$ ✓

$\therefore\ (x-1)^2 + (y-1)^2 = 1$

(2)

(ii) $\boldsymbol{v}(t) = cos(t)\boldsymbol{i} + sin(t)\boldsymbol{j}$ ✓

$\therefore\ speed = |\boldsymbol{v}(t)| = |cos(t)\boldsymbol{i} + sin(t)\boldsymbol{j}|$

$|\boldsymbol{v}(t)| = \sqrt{(cos(t))^2 + (sin(t))^2}$

$|\boldsymbol{v}(t)| = 1$ ✓

(2)

(iii) $\boldsymbol{r}(t) = (1+sin(t))\boldsymbol{i} + (1-cos(t))\boldsymbol{j}$

$\boldsymbol{r}(\pi) = \boldsymbol{i} + 2\boldsymbol{j}$ ✓

$\boldsymbol{v}(t) = cos(t)\boldsymbol{i} + sin(t)\boldsymbol{j}$

$\boldsymbol{a}(t) = -sin(t)\boldsymbol{i} + cos(t)\boldsymbol{j}$ ✓

$\boldsymbol{a}(\pi) = -sin(\pi)\boldsymbol{i} + cos(\pi)\boldsymbol{j}$

$\boldsymbol{a}(\pi) = -\boldsymbol{j}$ ✓

(3)

6. (9 marks)

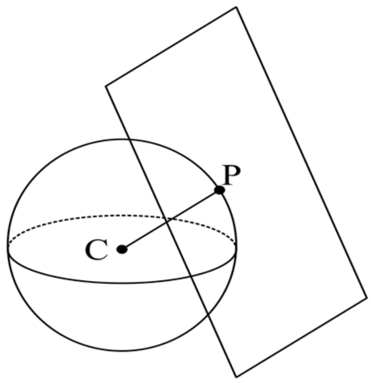
(a) (i) $r^2 = 2^2 + (-1)^2 + 4^2$

$r^2 = 21$

$r = \sqrt{21}$ ✓

Equation of sphere is

$(x-2)^2 + (y-3)^2 + (z+1)^2 = 21$ ✓



(ii) $\boldsymbol{CT} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$ ✓

Equation of the plane $P:\ 2x - y + 4z = d$ ✓

To get d :

$T(4,2,3) \quad 8 - 2 + 12 = d$

$d = 18$

$P:\ 2x - y + 4z = 18$ ✓

(3)

(iii) Two points are $(0,0,4.5)$ and $(9,0,0)$ ✓✓

Answers will vary but the points must lie on the plane $2x - y + 4z = 18$.

(2)

(b) $\mathbf{r} = \mathbf{OA} + s\,\mathbf{AB} + t\,\mathbf{AC}$

$$\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + s\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} + t\begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix} \quad ✓✓$$

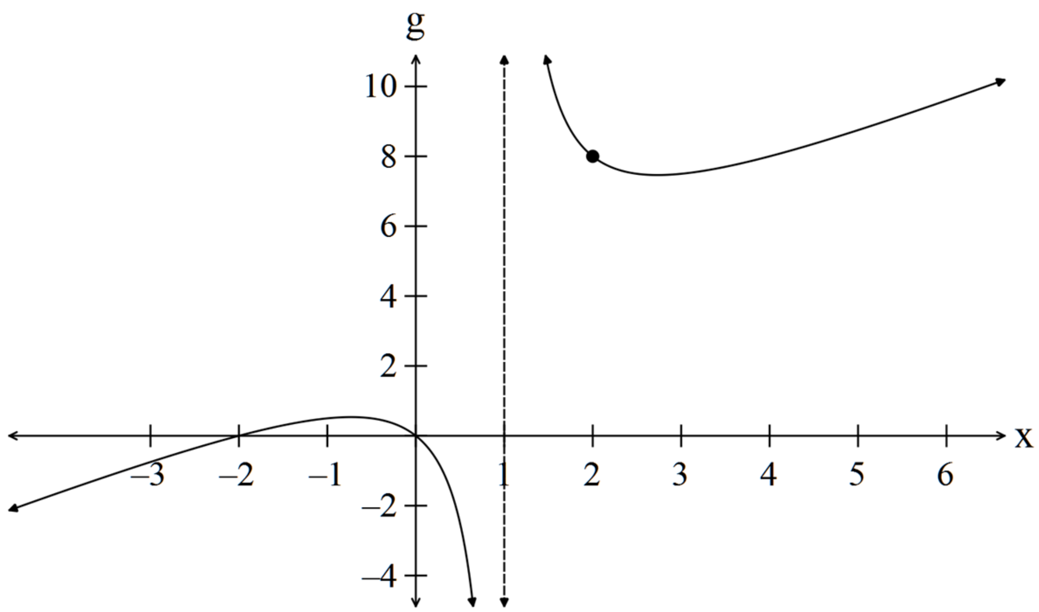
Could be any two of $k\,\mathbf{AB},\ l\,\mathbf{AC}$ or $m\,\mathbf{BC}$

Could be any one of $\mathbf{OA}, \mathbf{OB}$ or $\mathbf{OC}$

(2)

7. (4 marks)

(a) $g(x) = \dfrac{x(x+2)}{(x-1)}$



✓ Intercepts

✓ Lim as $x \to \pm\infty$

✓ L^- and L^+ at $x = 1$ \

✓ General shape and curvature -1/error

(4)

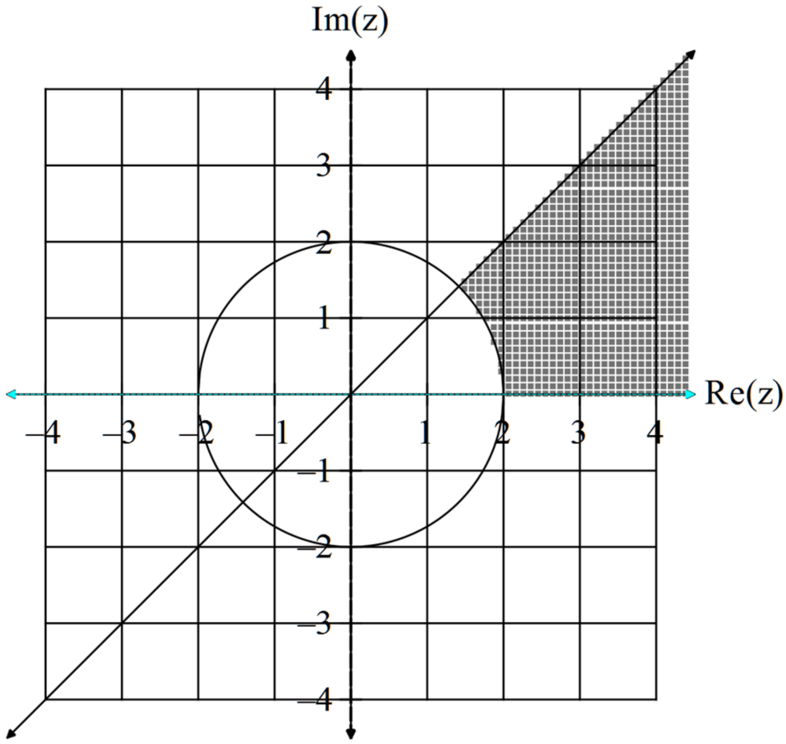
END OF SECTION ONE

Section Two

8. (6 marks)

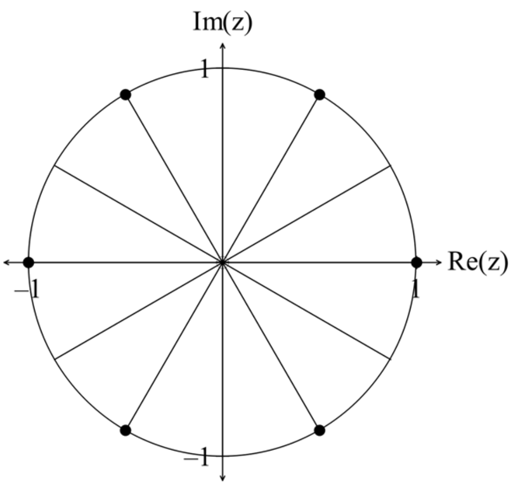
(a) Sketch the shaded region defined by

$$\left\{ z: |z| \geq 2 \ \cap \ 0 \leq Arg(z) \leq \frac{\pi}{2} \ \cap \ Im(z) \leq Re(z) \right\}$$



✓ Correct argument
✓ Correct line
✓ Outside circle -1/error (3)

(b) (i)



✓

(ii) $(1,0),\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right),\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right),(-1,0),\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right),\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$ ✓ ✓ -1/error

or

$(1,0),\left(1,\frac{\pi}{3}\right),\left(1,\frac{2\pi}{3}\right),(1,\pi),\left(1,-\frac{\pi}{3}\right),\left(1,-\frac{2\pi}{3}\right)$

(2)

9. (3 marks)

$\frac{v^2}{2}=\int(3-2x)\,dx$

$\frac{v^2}{2}=3x-x^2+c$ ✓

If $x=1, v=2$ $\quad \frac{4}{2}=3-1+c \Rightarrow c=0$ ✓

$\therefore \frac{v^2}{2}=3x-x^2$

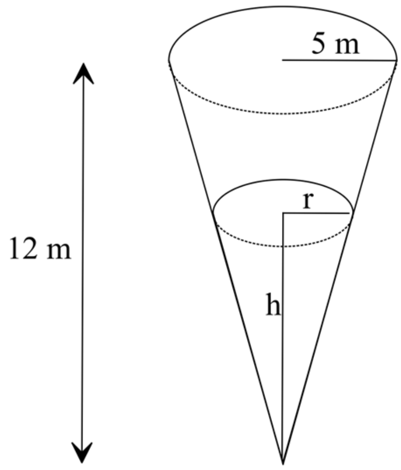
If $x=2, v=?$ $\quad \frac{v^2}{2}=6-4=2$

$v^2=4$

$v=\pm 2$ ✓

(3)

10. (8 marks)



(a) $\frac{dV}{dt} = 1.5m^3 / min$

$V = \frac{1}{3} \times \pi r^2 h$ $\qquad \frac{r}{h} = \frac{5}{12} \Rightarrow r = \frac{5h}{12}$ ✓

$V = \frac{\pi}{3} \times \left(\frac{5h}{12}\right)^2 h$

$V = \frac{25\pi h^3}{3 \times 144}$ ✓

Differentiate w.r.t. time t

$\frac{dV}{dt} = \frac{\cancel{3} \times 25\pi h^2}{\cancel{3} \times 144} \times \frac{dh}{dt}$ ✓

At $h = 2$

$1.5 = \frac{25\pi \times 2^2}{144} \times \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{54}{25\pi} m / min \quad = 0.688 m / min$ ✓

(4)

(b) $V = ?$ at $t = 2 min$

$V = 3m^3$

$V = \frac{25\pi h^3}{3 \times 144}$

$3 = \frac{25\pi h^3}{3 \times 144}$ ✓

$h^3 = \frac{9 \times 144}{25\pi}$

$h = 2.546$ ✓

(2)

(c) $r = \frac{5h}{12}$

$\frac{dr}{dt} = \frac{5}{12} \times \frac{dh}{dt}$ ✓

At $h = 2,\ \frac{dh}{dt} = \frac{54}{25\pi}$

$\therefore\ \frac{dr}{dt} = \frac{5}{12} \times \frac{54}{25\pi}$

$\frac{dr}{dt} = 0.286\,m/mi$ ✓

(2)

11. (16 marks)

(a) (i) $\int \frac{-3}{2p-1}\,dp = -\frac{3\ln(2p-1)}{2} + c$ ✓✓ -1/error (2)

(ii) $\int_{1.34}^{3.67} \frac{-3}{2p-1}\,dp = -1.992$ ✓✓ (2)

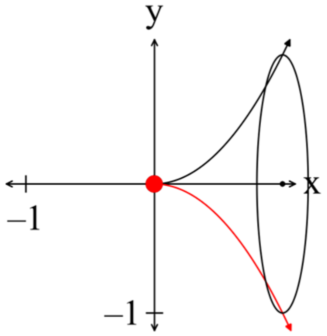
(b) Area = $\int_{\pi/4}^{5\pi/4} (\sin(x) - \cos(x)) = 2.828$

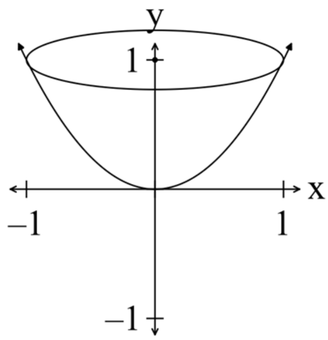
Bounds ✓

Function ✓

Answer ✓ (3)

(c)





$V_x = \int_0^1 \pi (y)^2\,dx$

$= \pi \int_0^1 x^4\,dx$ ✓

$= \frac{\pi}{5}\left[x^5\right]_0^1$

$= \frac{\pi}{5}(1-0)$ ✓

$V_x = \frac{\pi}{5} units^3$ ✓

$V_y = \int_0^1 \pi (x)^2\,dy$

$= \pi \int_0^1 y\,dy$ ✓

$= \frac{\pi}{2}\left[y^2\right]_0^1$

$= \frac{\pi}{2}(1-0)$

$V_y = \frac{\pi}{2} units^3$ ✓

Therefore $V_x \neq V_y$. (5)

(d) $y = 2x^2 - 0.5$ $\quad\quad$ $y = 2x^2 + 0.5$

If $x = 1, y = 1.5$ $\quad\quad$ If $x = 0, y = 0.5$ If $x = 1, y = 2.5$; ✓

$V = \int_0^{1.5} \pi(x)^2\, dy$ $\quad\quad$ $V = \int_{0.5}^{2.5} \pi(x)^2\, dy$

$= \pi\int_0^{1.5} \left(\frac{y+0.5}{2}\right) dy$ $\quad\quad$ $= \pi\int_{0.5}^{2.5} \left(\frac{y-0.5}{2}\right) dy$

$$\therefore V = \pi\int_0^{1.5} \left(\frac{y+0.5}{2}\right) dy \; + \; \pi\int_{1.5}^{2.5} 1\, dy \; - \; \pi\int_{0.5}^{2.5} \left(\frac{y-0.5}{2}\right) dy$$

✓ ✓ ✓ (4)

12. (3 marks)

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$\frac{\delta V}{\delta r} \approx \frac{dV}{dr}$

$\delta V \approx \frac{dV}{dr} \times \delta r$ ✓

$At\ r = 1.5\,cm$

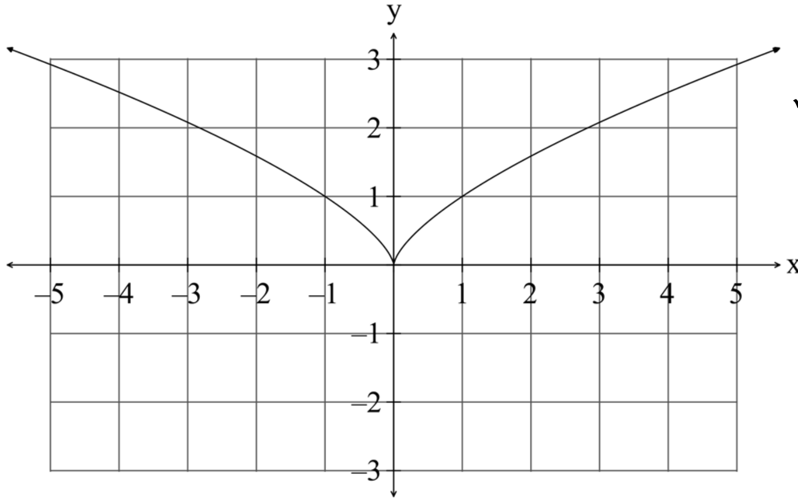
$\delta V \approx 4\pi \times 1.5^2 \times \frac{0.01}{10}$ ✓

$\delta V \approx 0.02827\ cm^3$ ✓

The increase in the volume is $0.02827\ cm^3$. (3)

13. (18 marks)

(a) (i) $f(g(x)) = f\left(x^{1/3}\right) = x^{2/3}$ ✓



✓✓

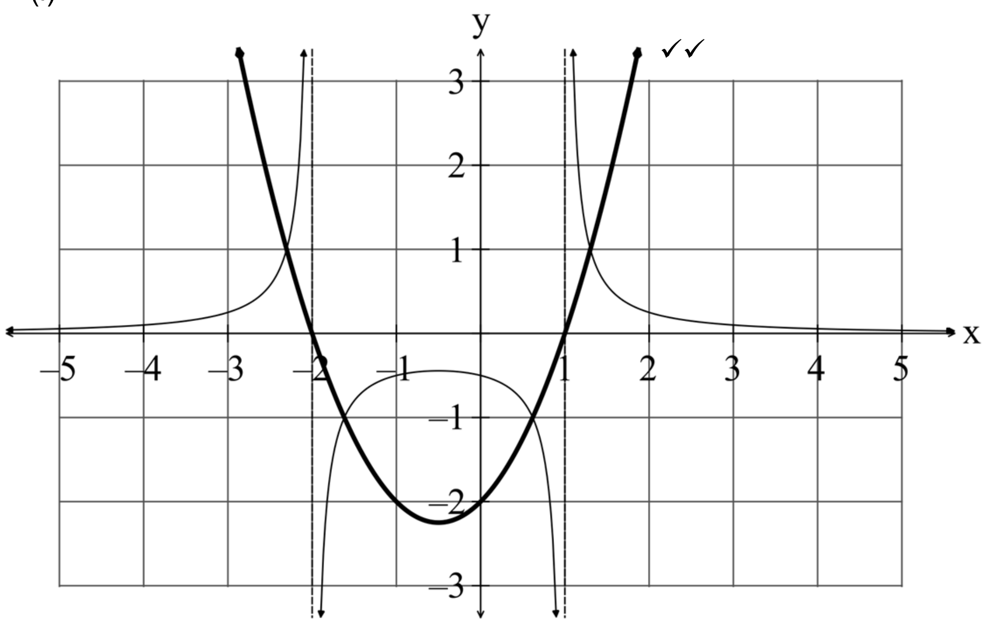
(3)

(ii) There is no inverse function as the function $y = f\left(g\left(x\right)\right)$ is not a one to one function. For example if $\mathrm{y} = 1, \mathrm{x} = \pm 1.$ ✓✓

(2)

(iii) $h\left(g\left(x\right)\right) = ln\left(x^{1/3}\right)$ ✓ $x > 0$ ✓ (2)

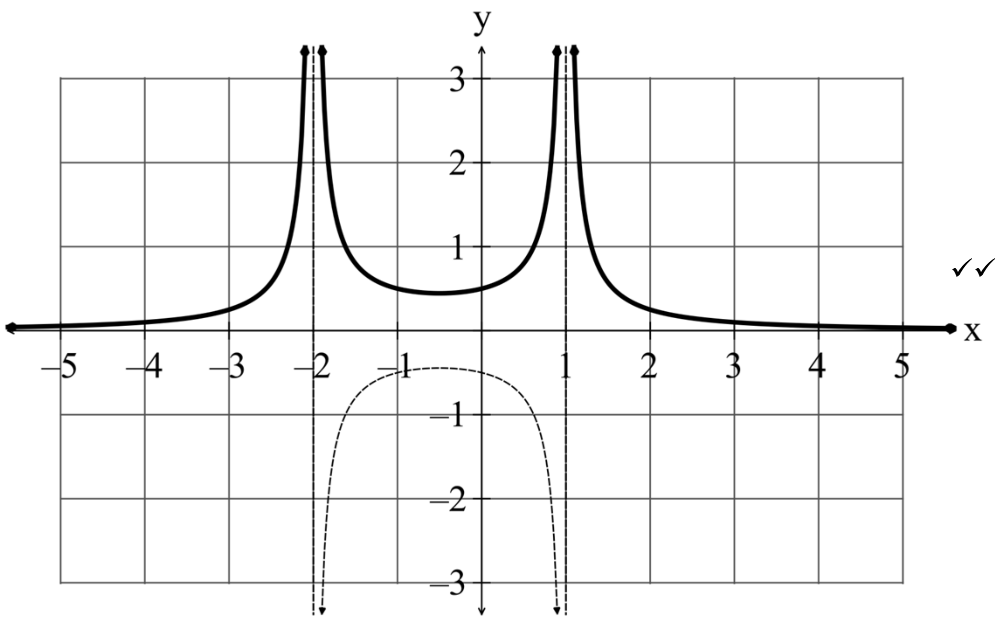
(b) (i)



(2)

(ii) $y = j\left(x\right) = \dfrac{1}{\left(x-1\right)\left(x+2\right)}$ ✓✓ (2)

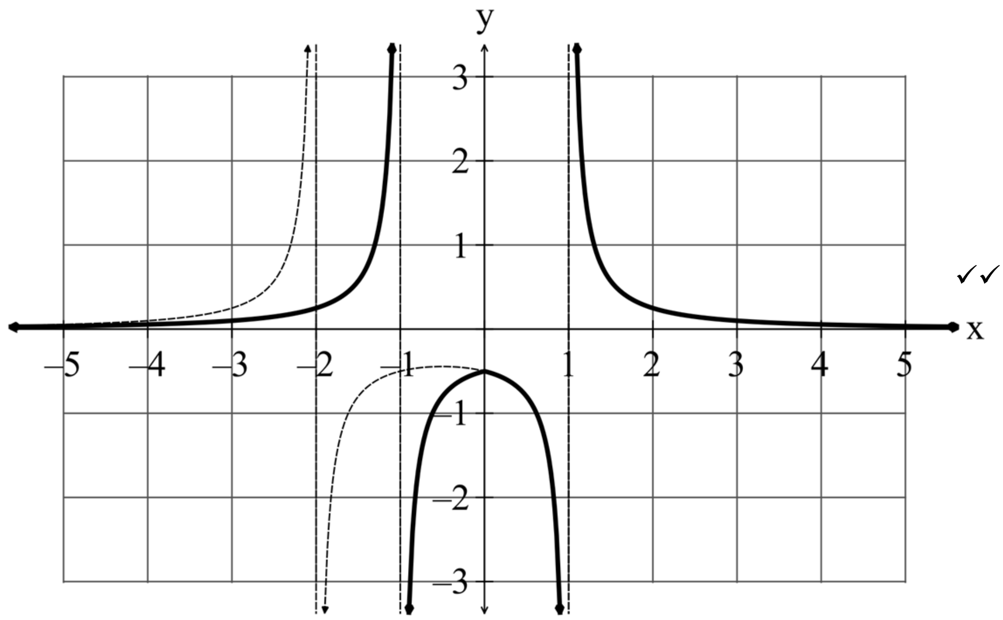
(iii) $y = \left|j\left(x\right)\right|$



✓✓

(2)

(iii) $y = j\left(|x|\right)$



✓✓

(2)

(c) $s\left(r\left(x\right)\right) = \dfrac{4x^2 - 4x + 1}{x^2}$

$= 4 - \dfrac{4}{x} + \dfrac{1}{x^2}$ ✓

$r\left(x\right) = \dfrac{1}{x}$ so $s\left(r\left(x\right)\right) = 4 - 4r\left(x\right) + \left(r\left(x\right)\right)^2$ ✓

$\therefore\ s\left(x\right) = 4 - 4x + x^2$ ✓

(3)

14. (6 marks)

(a) $\frac{dy}{dx} = -\frac{2xy}{1+x^2}$

$\frac{dy}{y} = -\frac{2x}{1+x^2}dx$

$\int \frac{dy}{y} = -\int \frac{2x}{1+x^2}dx$ ✓

$ln(y) = -ln(1+x^2) + c$

$c = ln(y) + ln(1+x^2)$ ✓

$c = ln((y)(1+x^2))$

$At\ (1,1)$

$c = ln((1)(1+1^2))$

$c = ln\,2$ ✓

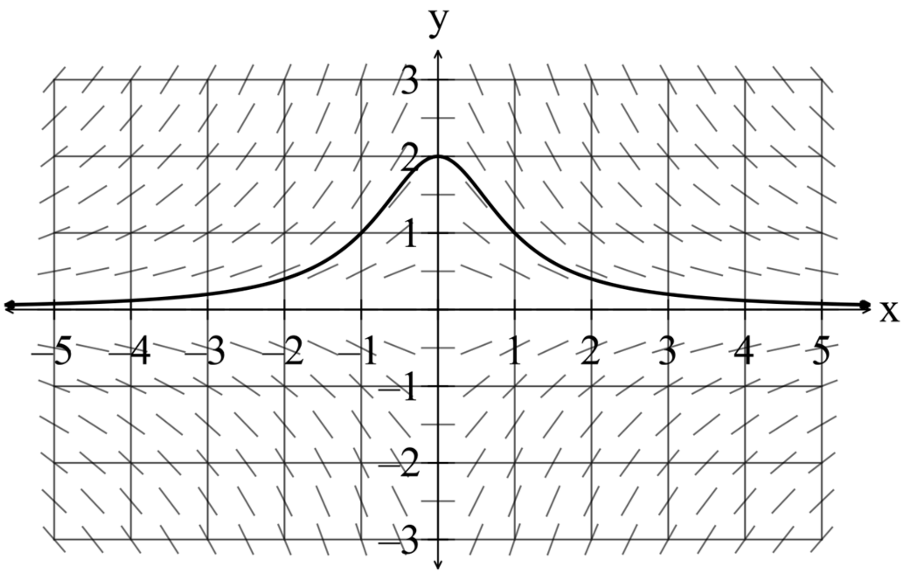
$ln(y) = -ln(1+x^2) + ln\,2$

$ln(y) = ln\left(\frac{2}{1+x^2}\right)$

$\therefore y = \frac{2}{1+x^2}$ ✓

(4)

(b)



✓✓

(2)

✓

15. (7 marks)

(a) $\boldsymbol{r}_s(t) = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \\ 0.5 \end{pmatrix}$

$\boldsymbol{r}_s(t) = \begin{pmatrix} 5 \\ 1+t \\ 0.5t \end{pmatrix}$ ✓

$B(6, 3, 0.5)$

$d^2 = (6-5)^2 + (3-(1+t))^2 + (0.5-0.5t)^2$ ✓

Minimum d^2 is at $(1.8, 1.2)$

$d^2 = 1.2$

$d = 1.095$ ✓

The salmon is a minimum of 1.095 m from the bear (so it escapes). (3)

(b) $\boldsymbol{r}_s(t) = \begin{pmatrix} 11 \\ 20 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 0.25 \\ 0 \end{pmatrix}$ ✓

$\boldsymbol{r}_E(t) = \begin{pmatrix} 3 \\ 5 \\ 80 \end{pmatrix} + t\begin{pmatrix} 2 \\ 4 \\ -20 \end{pmatrix}$ ✓

$\boldsymbol{r}_E(t) = \begin{pmatrix} 3+2t \\ 5+4t \\ 80-20t \end{pmatrix}$

If $80 - 20t = 0$, $t = 4$

It takes 4 seconds for the eagle to reach ground level. ✓

$At\ t = 4$, $\boldsymbol{r}_s(4) = \begin{pmatrix} 11 \\ 20 \\ 0 \end{pmatrix} + 4\begin{pmatrix} 0 \\ 0.25 \\ 0 \end{pmatrix} = \begin{pmatrix} 11 \\ 21 \\ 0 \end{pmatrix}$

$\boldsymbol{r}_E(4) = \begin{pmatrix} 3+8 \\ 5+16 \\ 0 \end{pmatrix} = \begin{pmatrix} 11 \\ 21 \\ 0 \end{pmatrix}$

The salmon and the eagle are at the same place at the same time.
Yes, the eagle catches the salmon. ✓

(4)

16. (86 marks)

(a)

$2x+3y-z=15$

$x+y+z=9$

$2x-y-z=3$

$$\begin{bmatrix} 1 & 1 & 1 & 9 \\ 2 & 3 & -1 & 15 \\ 2 & -1 & -1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & -1 & 3 & 3 \\ 0 & 3 & 3 & 15 \end{bmatrix} \quad \begin{matrix} \\ 2R_1 - R_2 \\ 2R_1 - R_3 \end{matrix}$$ ✓

$$\begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & -1 & 3 & 3 \\ 0 & 1 & 1 & 5 \end{bmatrix} \quad R_3 \div 3$$

$$\begin{bmatrix} 1 & 1 & 1 & 9 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & 4 & 8 \end{bmatrix} \quad \begin{matrix} \\ -R_2 \\ R_2 + R_3 \end{matrix}$$ ✓

$4z=8 \Rightarrow z=2$

$y-3z=-3$

$y-6=-3 \Rightarrow y=3$ ✓

$x+y+z=9$

$x+3+2=9 \Rightarrow x=4$

$\therefore (4,3,2)$ is the point of intersection. ✓

(4)

(b)

$2x+3y-z=5$

$-2x-3y+z=-15$

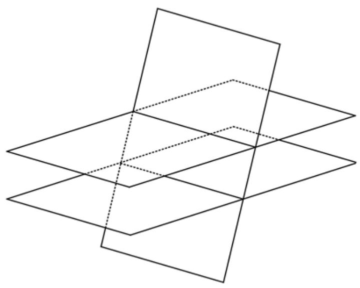
$x+y+z=9$

$P_1 \quad 2x+3y-z=5$

$P_2 \quad -2x-3y+z=-15$

i.e. $P_2 \quad 2x+3y-z=15$ ✓

Which means P_1 is parallel to P_2 so there are no solutions. ✓ (2)



17. (8 marks)

(a) (i) The sampling distribution is normally distributed. ✓✓ (2)

(ii) Each sample has a mean. The distribution of the means approximates the true mean and the distribution has a small standard deviation because the mean of each sample approximates the mean itself.
The average of the means is a good estimate of the true mean. ✓✓ (2)

(iii) Each sample has a mean that approximates the true mean.
The mean of the samples, used as a sampling distribution, reduces the variation of the sampling points. The means of the samples would be similar to each other, so the standard deviation of the sampling distribution will have a very small standard deviation compared to the parent population. ✓✓ (2)

(b) $\mu = 20$ ✓

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

$1.5 = \frac{\sigma}{\sqrt{100}}$

$1.5 \times 10 = \sigma$

$\sigma = 15$

$\therefore\ \sigma^2 = 225$ ✓

(2)

18. (8 marks)

(a) $4.2 - 1.96 \times \frac{0.5}{\sqrt{16}} \le \mu \le 4.2 + 1.96 \times \frac{0.5}{\sqrt{16}}$ ✓

$4.2 - 0.245 \le \mu \le 4.2 + 0.245$

$3.96 \le \mu \le 4.45$
✓ ✓

The biologist can be 95% confident that the average weight of the Chandler Road adult male quokkas is between ✓3.96 and 4.45 kgs.
If the sampling is repeated, 95% of the confidence intervals contain the mean weight.
✓

(4)

(b) $\text{d} = 0.2\,\text{kg}, \quad \text{z} = 2.576, \quad \sigma = 0.5 \text{ kg}$

Using $\text{d} = \text{z} \times \left(\frac{\sigma}{\sqrt{\text{n}}} \right)$,

$$\text{n} = \frac{\text{z}^2 \times \sigma^2}{\text{d}^2} \quad ✓$$

$$\text{n} = \frac{2.576^2 \times 0.5^2}{0.2^2}$$

$$\text{n} = 41.4736 \quad ✓$$

✓

The biologist would need to weigh 42 adult male quokkas to be 99% sure the mean weight was within 0.2 kilograms of the true mean.

. (4)

19. (10 marks)

(a) $P = \frac{a}{1 + be^{-kt}}$

$$\frac{dP}{dt} = -a\left(1 + be^{-kt}\right)^{-2} \times be^{-kt} \times (-k) \quad ✓$$

$$\frac{dP}{dt} = \frac{abk \times e^{-kt}}{\left(1 + be^{-kt}\right)^2} \quad ✓$$

but $a > 0, b > 0$ and $k > 0$

$e^{-kt} > 0$ and $\left(1 + be^{-kt}\right)^2$ ✓

so $\frac{abke^{-kt}}{\left(1 + be^{-kt}\right)^2} > 0$

$\therefore \frac{dP}{dt} > 0$

(3)

(b) (i) $P = \dfrac{a}{1+be^{-kt}}$

$3000 = \lim_{t\to\infty}\left(\dfrac{a}{1+be^{-kt}}\right)$ ✓

$\lim_{t\to\infty}\left(e^{-kt}\right) = \lim_{t\to\infty}\left(\dfrac{1}{e^{kt}}\right) = 0$

$\therefore a = 3000$ ✓

$\therefore\ P = \dfrac{3000}{1+be^{-kt}}$

$At\ t = 0,\ \ P = 50$

$50 = \dfrac{3000}{1+be^{-k\times 0}} \quad and\ \ e^{-k\times 0} = 1$ ✓

$50(1+b) = 3000$

$b = 59$ ✓

$\therefore\ P = \dfrac{3000}{1+59e^{-kt}}$

$At\ t = 5,\ \ P = 215$

$so\ \ 215 = \dfrac{3000}{1+59e^{-k\times 5}}$

$k = 0.3032344639$ ✓

(5)

(ii) $At\ \ t = 15,\ P = ?$

$\therefore\ P = \dfrac{3000}{1+59e^{-0.3032344639t}}$

$P \approx 1847$ ✓

(1)

END OF SECTION TWO